5.9 Binary modulated bandpass signal

Binary modulated bandpass signal

- For digital modulated signals, the modulating signal m(t) is a digital signal given by the binary or multilevel line codes

- The most common binary bandpass techniques are as follows:
  - On-off keying (OOK)
  - Binary phase-shift keying (BPSK)
  - Frequency-shift keying (FSK)

OOK

OOK

OOK signal:

$$s(t) = A_c m(t)\cos\omega_c t$$

complex envelope:

$$g(t) = A_c m(t)$$

where *m*(*t*) is the unipolar baseband data signal. It is shown that OOK is identical to unipolar binary modulation on a DSB-SC signal.

PSD

the PSD of the line codes:

$$P_s(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$

$$P_g(f) = \frac{A_c^{\,2}}{2}\left[\delta(f) + T_b\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2\right]$$

OOK

Bit rate is $R=1/T_b$

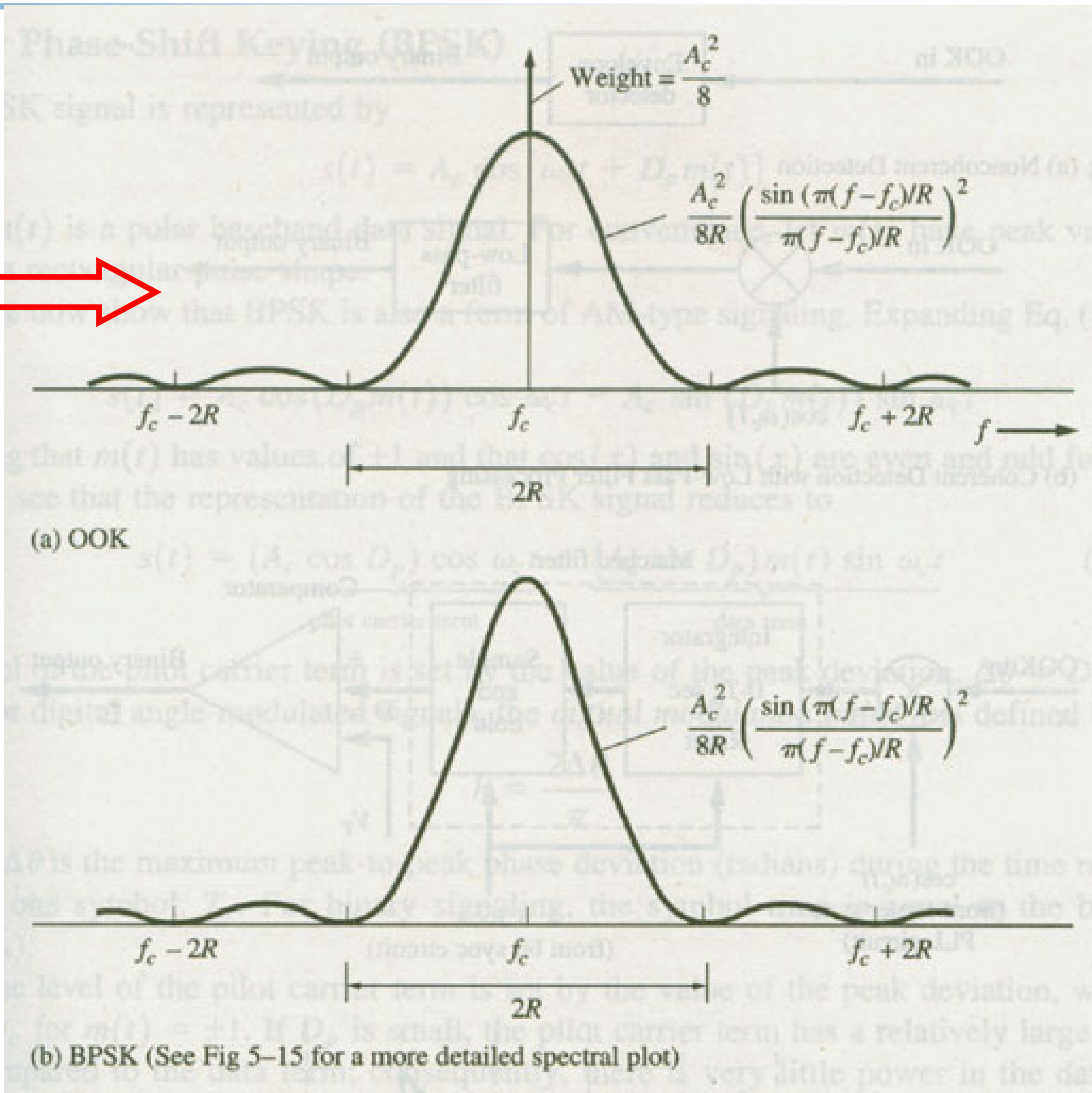
The **null-to-null bandwidth** is $2R$

the **absolute bandwidth** is ∞

The transmission bandwidth of the OOK signal is

$B_T=2B$

B is the baseband bandwidth



OOK

If raised cosine-rolloff filtering is used, the absolute baseband bandwidth is

$$D = \frac{2B}{1+r} \Rightarrow B = \frac{1}{2}(1+r)R$$

where r is the rolloff factor of the filter

The absolute transmission bandwidth is

$$B_T = 2B = (1+r)R$$

Demodulation for OOK

1) Envelope detector (noncoherent detection)

2) product detector (coherent detection) with Low-pass Filter Processing

3) product detector (coherent detection) with Matched Filter Processing.

BPSK

BPSK

Bandpass signal for BPSK:

$$s(t) = A_c \cos[\omega_c t + \theta(t)]$$
$$\theta(t) = D_P m(t)$$

$$\Rightarrow \quad s(t) = A_c \cos[\omega_c t + D_p m(t)]$$

Expanding equation above we get:

$$s(t) = A_c \cos(D_p m(t)) \cos\omega_c t - A_c \sin(D_p m(t)) \sin\omega_c t$$
$$= A_c \cos(D_P) \cos\omega_c t - A_c \sin(D_P) m(t) \sin\omega_c t$$

pilot carrier term **data term**

What is the meaning of *D_P*?

$$\Delta\theta = D_P \max[m(t)] = D_P$$

Definition: ***the*** **digital modulation index** $h = \dfrac{2\Delta\theta}{\pi}$

BPSK

To maximize the signaling efficiency (so that there is a low probability of error), the power in the data term needs to be maximized.

Let the digital modulation index:

$$h = \frac{2\Delta\theta}{\pi} = \frac{2D_p}{\pi} = 1$$

Which corresponds to $\Delta\theta = D_p = \pi / 2$

The BPSK signal becomes:

$$s(t) = A_c \cos[\omega_c t + D_p m(t)]$$
$$= -A_c m(t) \sin \omega_c t$$

it equivalent to DSB-SC signaling with a polar baseband data waveform

BPSK

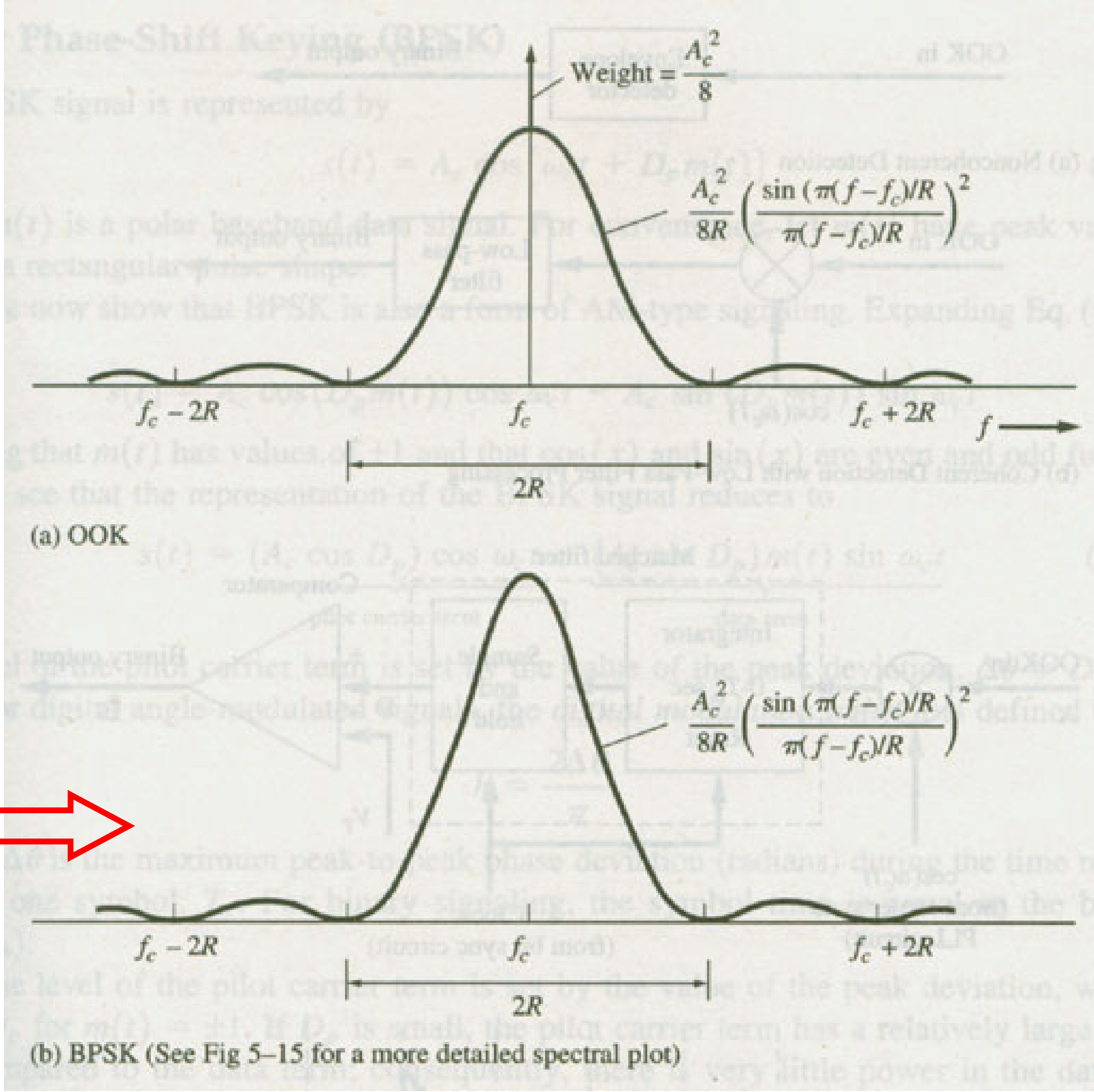
Conclusion:

For this optimum BPSK signal of $h=1$, the complex envelope is

$$g(t) = jA_c m(t)$$

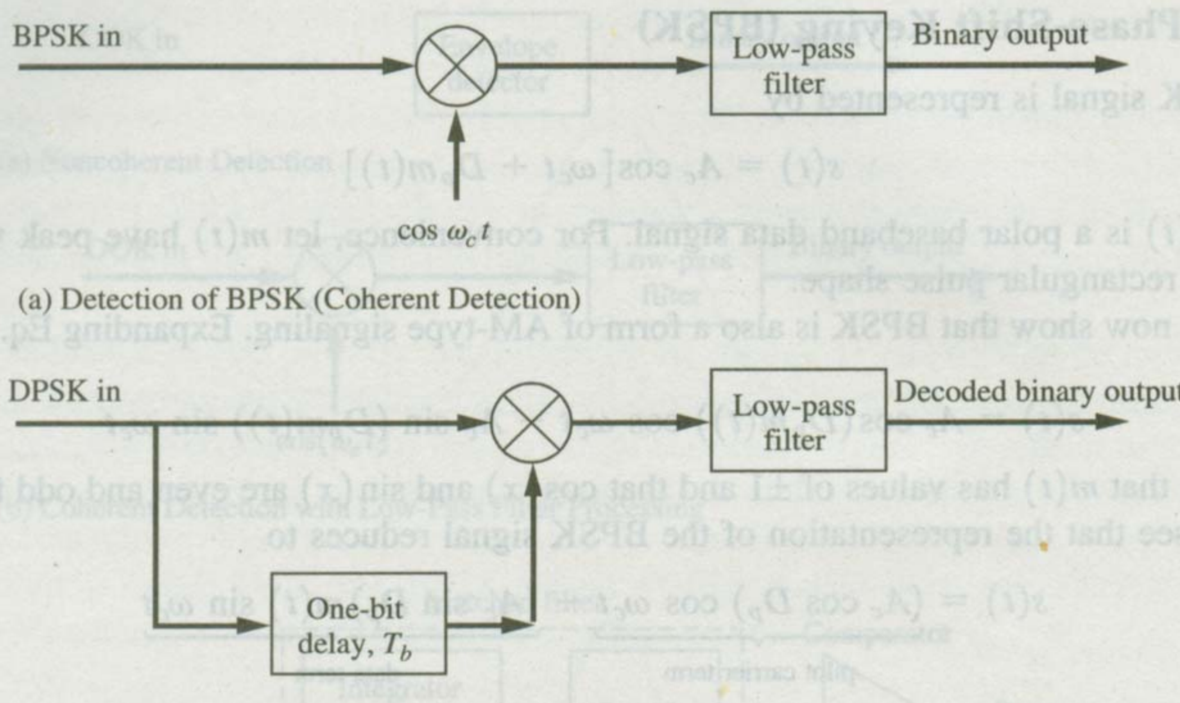
PSD of complex envelope:

$$P_g(f) = A_c^2 T_b \left(\frac{\sin \pi f T_b}{\pi f T_b} \right)^2$$



Detection of BPSK

Because there is no discrete carrier term in the BPSK signal, synchronous detection must be used to detect BPSK.



(a) Detection of BPSK (Coherent Detection)

(b) Detection of DPSK (Partially Coherent Detection)

Figure 5–22 Detection of BPSK and DPSK.

DPSK

DPSK: the data on the BPSK signal are differentially encoded, the decoded data sequence will be recovered at the output of the receiver. This signaling technique consisting of transmitting a differentially encoded BPSK signal is known as DPSK.

Goal: to solve the problem that the phase-shift-keyed signals cannot be detected incoherently

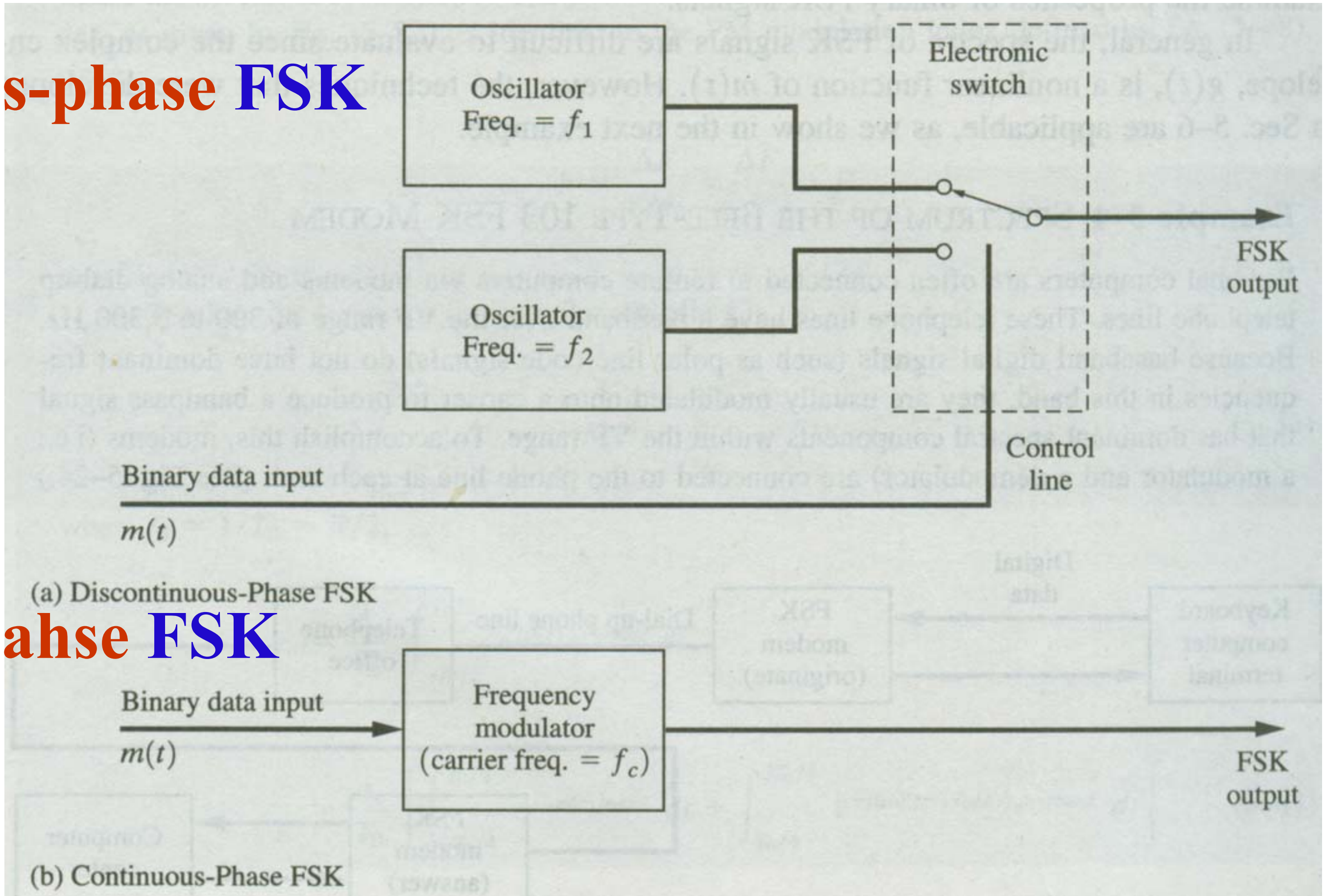
Way: partially coherent technique to be used

FSK

Generation of FSK

Two different types of FSK

- Discontinuous-phase **FSK**



- Continuous-pahse **FSK**

Figure 5–23 Generation of FSK.

FSK

Discontinuous-phase FSK signal is represented by

$$s(t) = A_c \cos[\omega_c t + \underline{\theta(t)}] = \begin{cases} A_c \cos(\omega_1 t + \theta_1) & \text{for } 1 \\ A_c \cos(\omega_2 t + \theta_2) & \text{for } 0 \end{cases}$$

The discontinuous phase function is

$$\theta(t) = \begin{cases} \omega_1 t + \theta_1 - \omega_c t & \text{for } 1 \\ \omega_2 t + \theta_2 - \omega_c t & \text{for } 0 \end{cases}$$

FSK

Continuous-phase FSK

$$s(t) = A_c \cos[\omega_c t + D_f \int_{-\infty}^{t} m(\lambda) d\lambda]$$

or

$$s(t) = \text{Re}\left\{ g(t) e^{j\omega_c t} \right\}$$

where

$$g(t) = A_c e^{j\theta(t)}$$

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda) d\lambda$$

Although *m*(t) is discontinuous at the switching time, the phase function θ(t) is continuous because θ(t) proportional to the integral of *m*(t).

FSK

Spectrum of FSK signals

$$S(f)=\frac{1}{2}[G(f-f_c)+G^*(-f-f_c)]$$

where

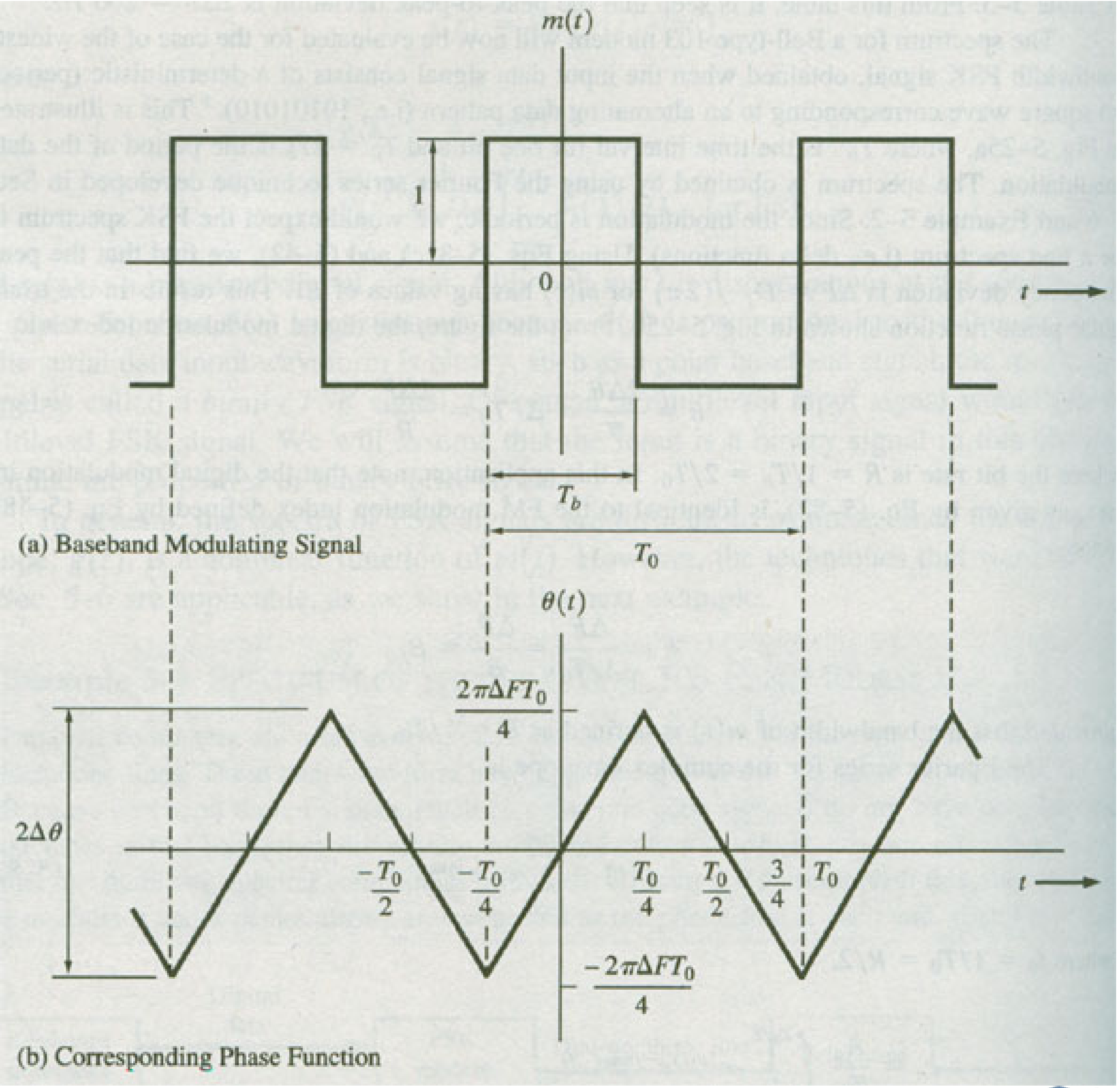
$$G(f)=F[g(t)]=F[A_c e^{j\theta(t)}]$$

The techniques that were developed in sec. 5-6 are applicable.

Example

5-4

Spectrum of the Bell-Type 103 FSK modem



FSK

Example 5-4 con.

g(t) is periodic with period $T_0=1/f_0$. Its Fourier series can be represented by:

$$g(t)=\sum_{n=-\infty}^{n=\infty} c_n e^{jn\omega_0 t}$$

where

$$c_n=\frac{A_c}{T_0}\int_{-T_0/2}^{T_0/2} e^{j\theta(t)} e^{-jn\omega_0 t}\,dt$$

which reduced to

$$c_n=\frac{A_c}{2}\left[\left(\frac{\sin[(\pi/2)(h-n)]}{(\pi/2)(h-n)}\right)+(-1)^n\left(\frac{\sin[(\pi/2)(h+n)]}{(\pi/2)(h+n)}\right)\right]$$

Example 5-4 con.

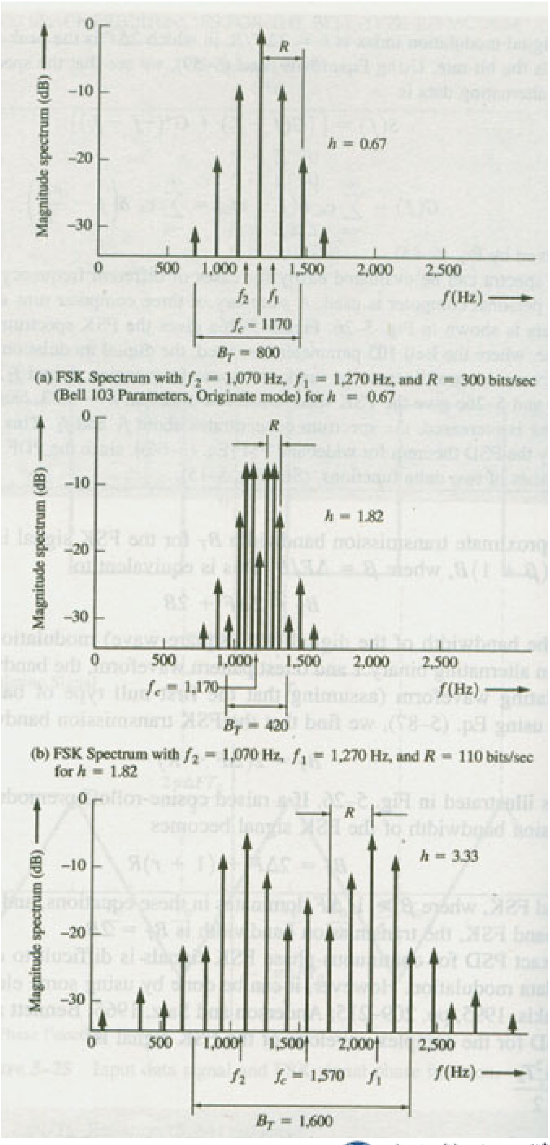
Taking the Fourier transform of *g(t)*, we get:

$$G(f) = \sum_{-n=\infty}^{n=\infty} c_n \delta(f - nf_0) = \sum_{-n=\infty}^{n=\infty} c_n \delta(f - nR/2)$$

Using the result in

$$S(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

We may get the spectrum of the FSK signal.



FSK

❖ **Bandwidth — Carson's rule:**

$$B_T = 2(\beta + 1)B$$

$$\beta = \Delta F / B$$

$$B_T = 2\Delta F + 2B$$

- **When modulating waveform is square-wave, $B=R$, the FSK transmission bandwidth becomes:**

$$B_T = 2(\Delta F + R)$$

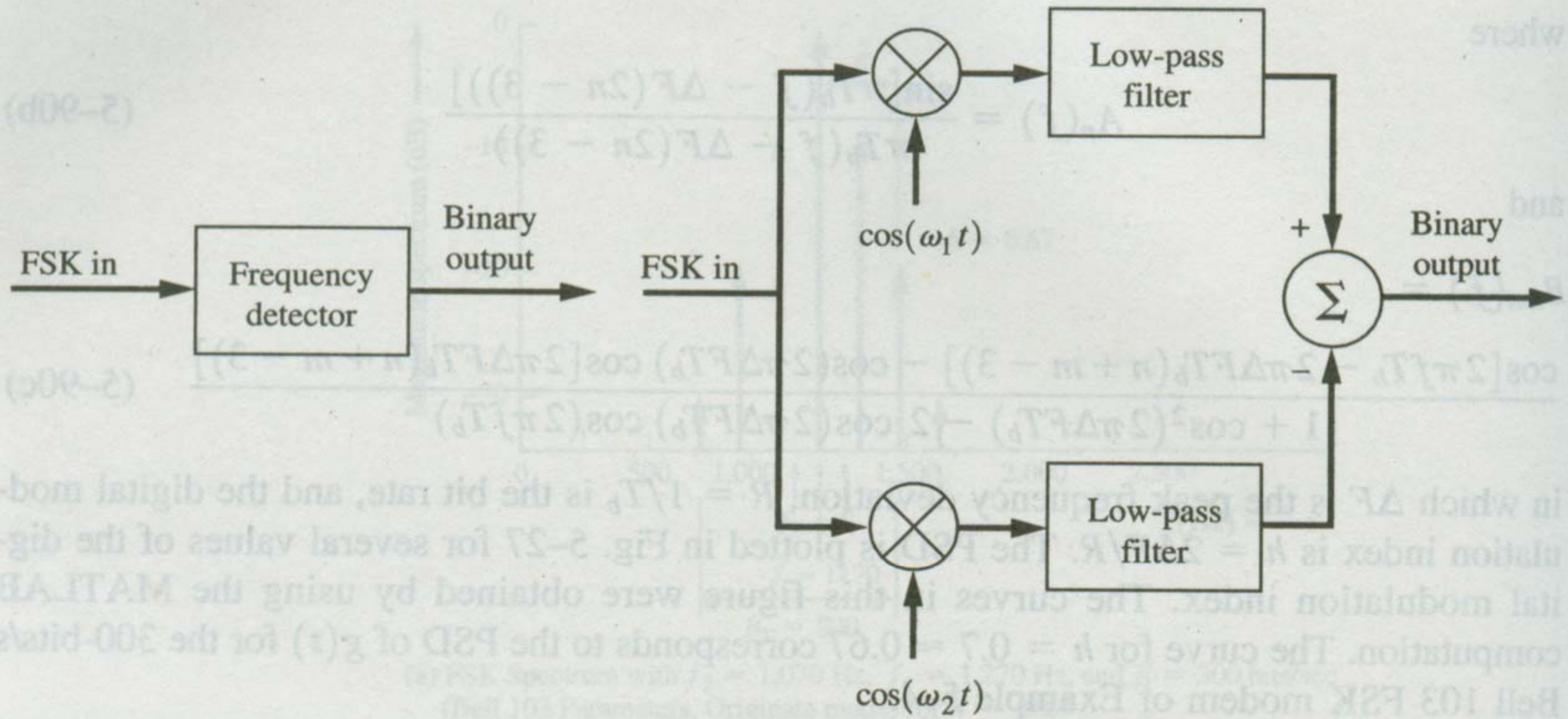
- **If a raised cosine-rolloff premodulation filter is used, $B=(1+R)/2$, the transmission bandwidth of the FSK signal becomes:**

$$B_T = 2\Delta F + (1+r)R$$

FSK

Detection

- Noncoherent detection
- Coherent detection



(a) Noncoherent Detection (b) Coherent (Synchronous) Detection

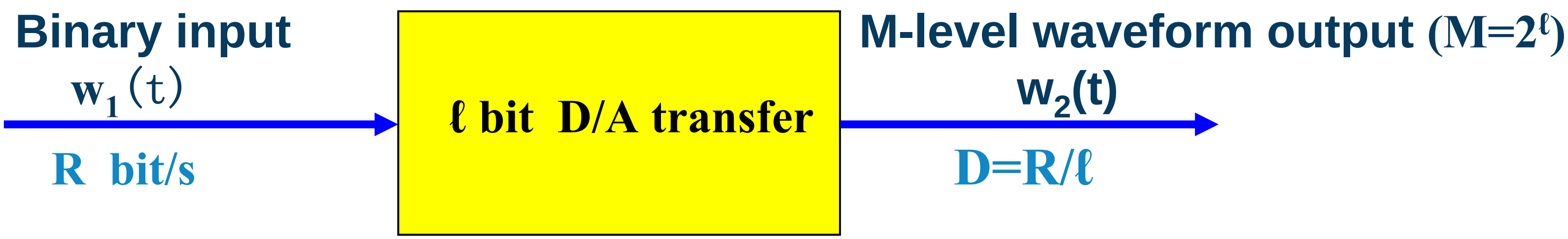
Figure 5–28 Detection of FSK.

5.10 Multilevel Modulated Bandpass Signaling

Multilevel Modulation

Generation of the multilevel signal

ℓ bit D/A transfer:



If a level stands for ℓ bits binary digits, the number of levels in the multilevel signal is : $M=2^{\ell}$, The symbol rate(baud) of the multilevel signal is $D=R/\ell$.

MPSK & QPSK

MPSK & QPSK

- For MPSK, the modulating signal m(t) is a M level digital signal ($M=2^{\ell}$).

MPSK signal :

$$s(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

complex envelope :

$$g(t) = A_c e^{j\theta(t)}$$

The permitted values of $\theta(t)$ are:

$$\theta_i(t) = \frac{2\pi i}{M} + \theta_0 \qquad i = 1, 2, \ldots M$$

Multilevel Modulation

M-ary phase-shift keying(MPSK) with M=4 is called QPSK .

QPSK signal constellations

(permitted values of the complex envelope)

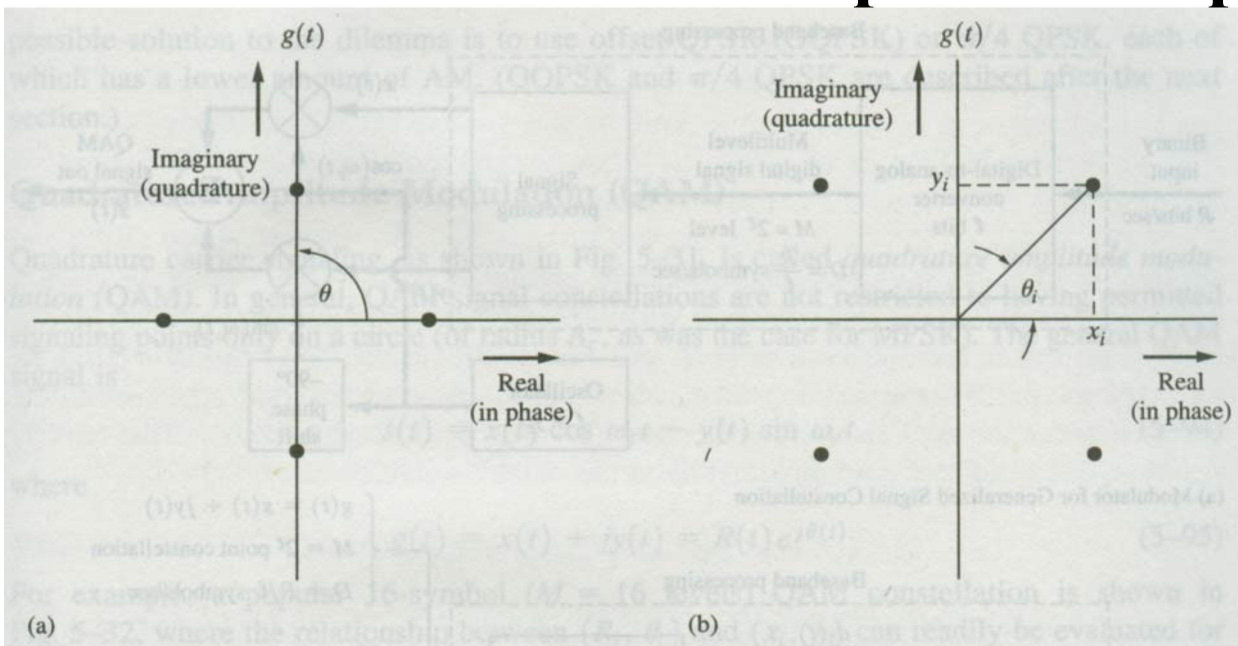


Figure 5–30 QPSK and $\pi/4$ QPSK signal constellations (permitted values of the complex envelope).

Multilevel Modulation

Generation of MPSK signal

Method 1: **Using a phase modulator**

Method 2: **Using a quadrature modulator**

in the method 2:

$$g(t) = A_c e^{j\theta(t)} = x(t) + jy(t)$$

Where the permitted values of x and y are:

$$\begin{cases} x_i = A_c \cos\theta_i \\ y_i = A_c \sin\theta_i \end{cases}$$

$\theta_i, i = 1, 2, \cdots, M$ **is the permitted phase angles of the MPSK signal.**

$$s(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

Multilevel Modulation

MPSK & QPSK

- For **rectangular-shape data pulse**, the envelope of the QPSK signal is constant, there is **no AM on the signal**.

- But the rectangular-shaped data **produce a (sin$x$/x)^2-type power spectrum** for QPSK signal that **has large undesirable spectral sidelobes.**

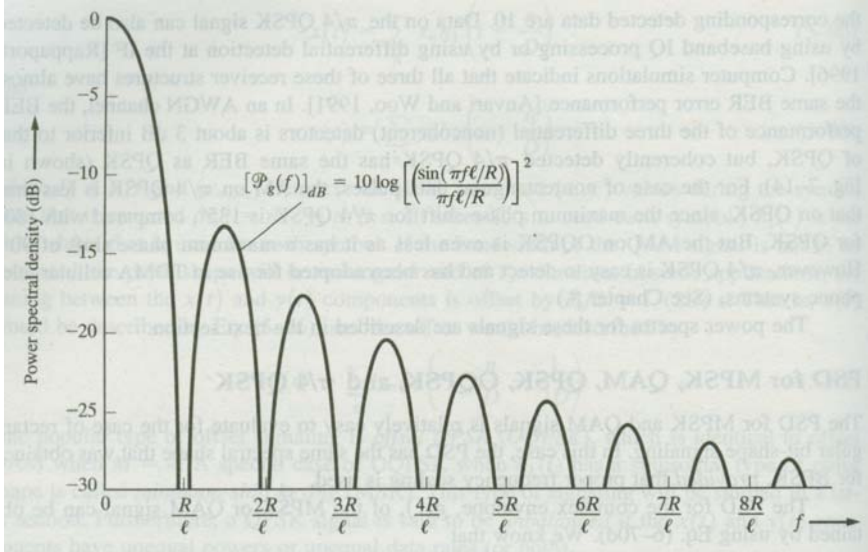


Figure 5–33 PSD for the complex envelope of MPSK and QAM with rectangular data pulses, where $M = 2^\ell$, R is the bit rate, and $R/\ell = D$ is the baud rate (positive frequencies shown). Use $\ell = 2$ for PSD of QPSK, OQPSK, and $\pi/4$ QPSK complex envelope.

Multilevel Modulation

MPSK & QPSK

- **The undesirable sidelobes can be eliminated if the data pulses are filtered to a pulse shape corresponding to a raised cosine-rolloff filter**

- **Unfortunately, although filtering solves the problem of poor spectral sidelobes, it creates another one:**

AM on the QPSK signal.

Solution: **OQPSK or π /4 QPSK be used.**

OQPSK & π/4 QPSK

OQPSK & π/4 QPSK

1. OQPSK

- OQPSK is the special QPSK, the I and Q conponent are offset by a 1/2 symbol interval (i.e., by 1 bit)

- The AM is reduced because a maximum phase transition of only π/2 occurs (as opposed to for QPSK)

2. π/4 QPSK

A π/4 QPSK signal is generated by alternating between two QPSK constellations that are rotated by π/4 with respect to each other.

Multilevel Modulation

OQPSK & π /4 QPSK

π / 4 QPSK

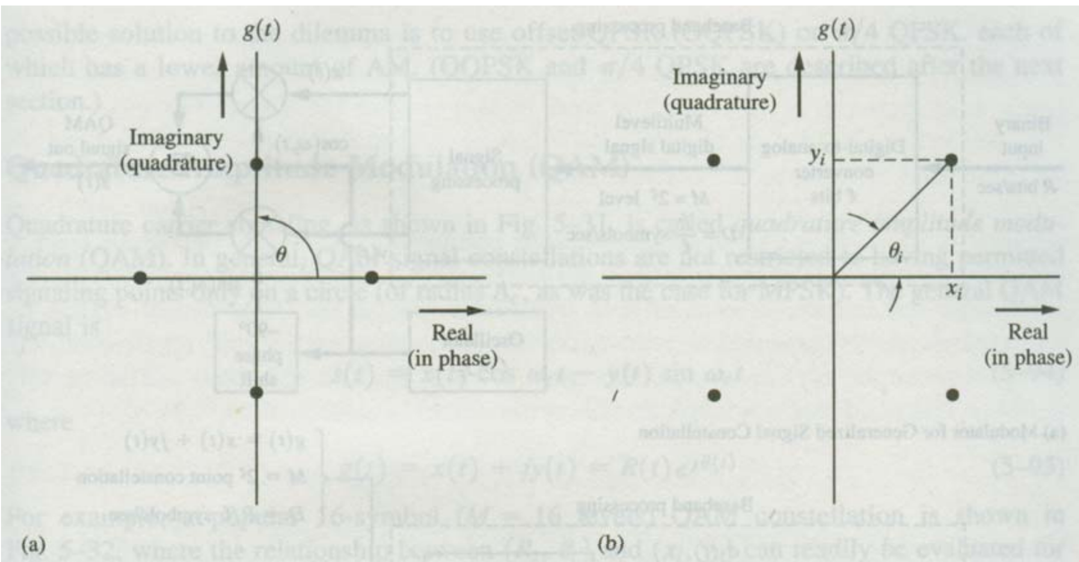
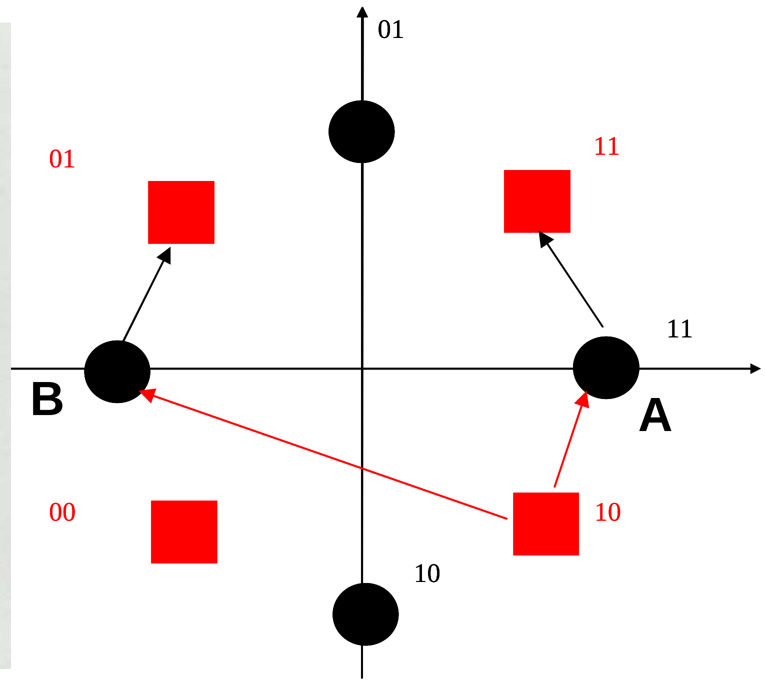


Figure 5–30 QPSK and $\pi/4$ QPSK signal constellations (permitted values of the complex envelope).



The two signal constellations are used alternately as follows:

Given a point on one of the signal constellations that corresponds to two bits of input data, two new bits are read to determine the next point that is selected from the other constellation.

Multilevel Modulation

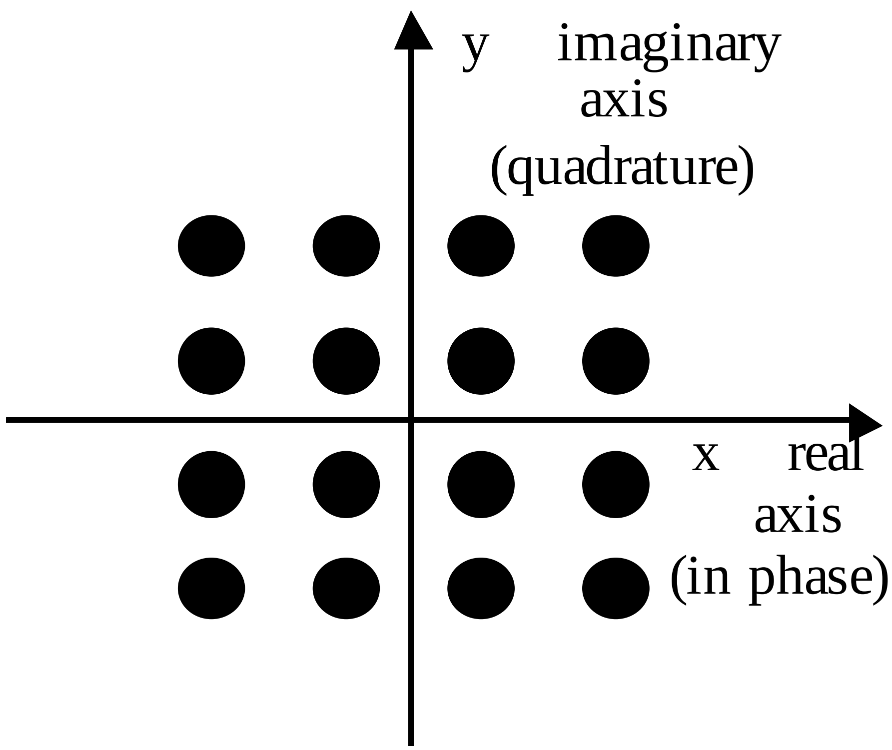
Detection of the $\pi/4$ QPSK signal:

- FM detector
- Baseband IQ processing
- Differential detection at IF

QAM

QAM

in general, the QAM signal constellations are **not restricted** to have permitted signaling points **only on a circle.**



Multilevel Modulation

QAM

The general QAM signal is:

$$s(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

its complex envelope is

$$g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}$$

where

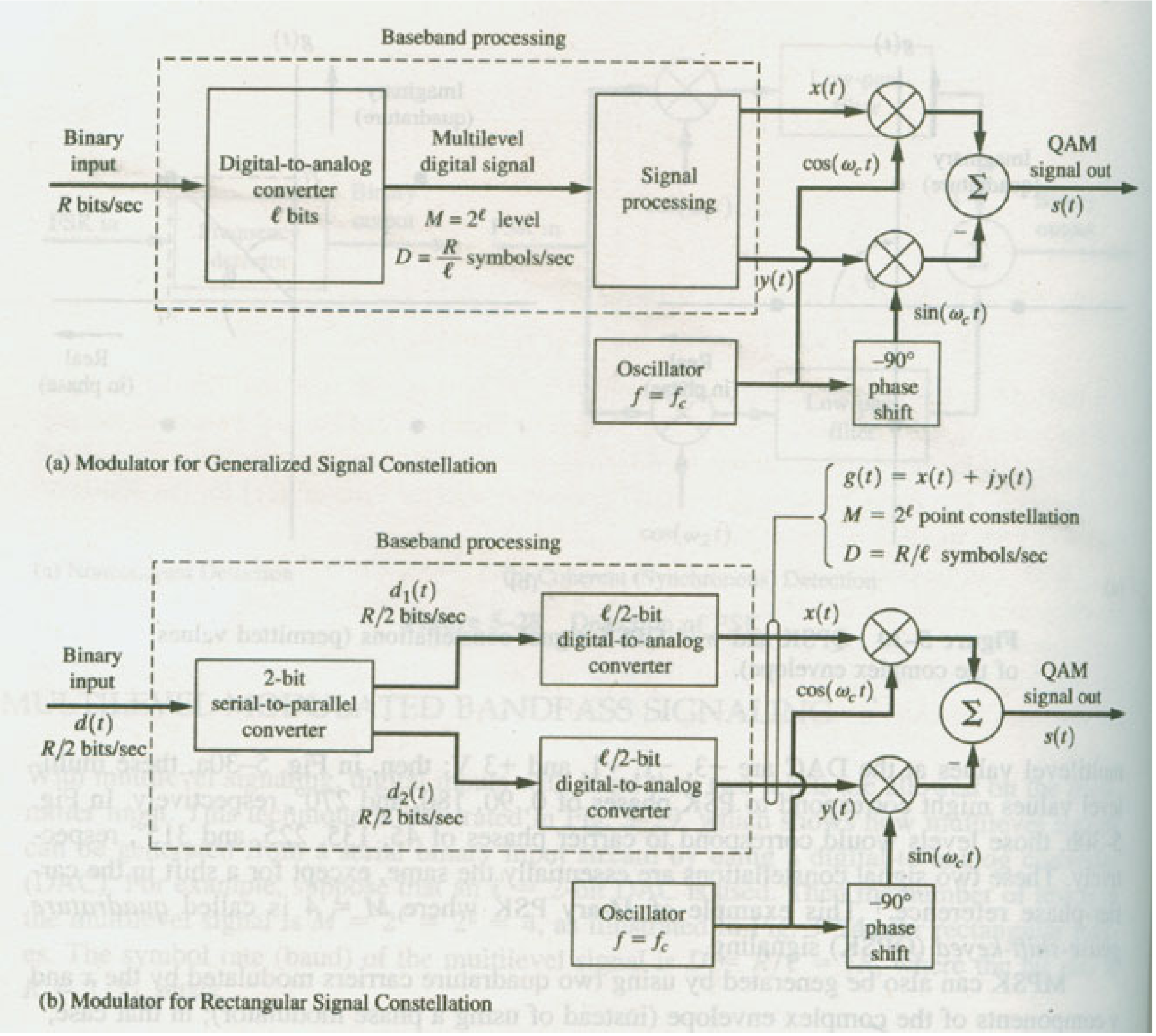
$$R(t) = \sqrt{x^2(t) + y^2(t)}$$

$$\theta(t) = \tan^{-1}(y(t)/x(t))$$

Multilevel Modulation

QAM

Generation of QAM signal



Multilevel Modulation

QAM

- The **waveforms of *I* and *Q* components** are represented by:

$$x(t)=\sum_n x_n h(t-\frac{n}{D})\qquad y(t)=\sum_n y_n h(t-\frac{n}{D})$$

where $h(t)$ is the pulse shape that is used for each symbol

In some applications, the timing between the $x(t)$ and $y(t)$ components is offset by $T_s/2=1/(2D)$, that is:

$$y(t)=\sum_n y_n h(t-\frac{n}{D}-\frac{1}{2D})$$

One popular type of offset signaling is OQPSK

Multilevel Modulation

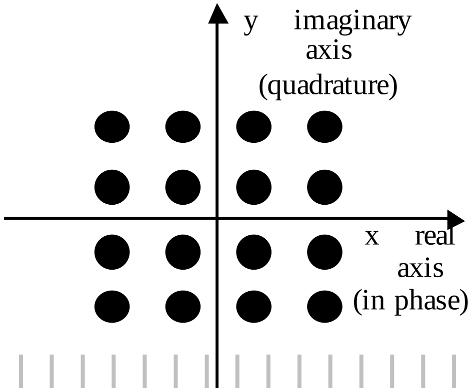
PSD for MPSK,QAM,QPSK,OQPSK and π /4 QPSK

The complex envelope for MPSK and QAM is represented by:

$$g(t) = \sum_{-\infty}^{\infty} c_n f(t - nT_s)$$

The rectangular pulse has the Fourier transform:

$$F(f) = T_s\left(\frac{\sin \pi f T_s}{\pi f T_s}\right) = \ell T_b\left(\frac{\sin \ell \pi f T_b}{\ell \pi f T_b}\right)$$



The mean value of c_n is: $m_c = \overline{c_n} = 0$

The variance is: $\sigma_c^2 = \overline{c_n c_n^*} = |c_n|^2 = C$

PSD & Spectral efficiency

Multilevel Modulation

PSD for MPSK,QAM,QPSK,OQPSK and π /4 QPSK

The **PSD for the complex envelope** of MPSK or QAM signal with data modulation of **rectangular bit shape** is

$$P_g(f) = K\left(\frac{\sin \pi f \ell T_b}{\pi f \ell T_b}\right)^2$$

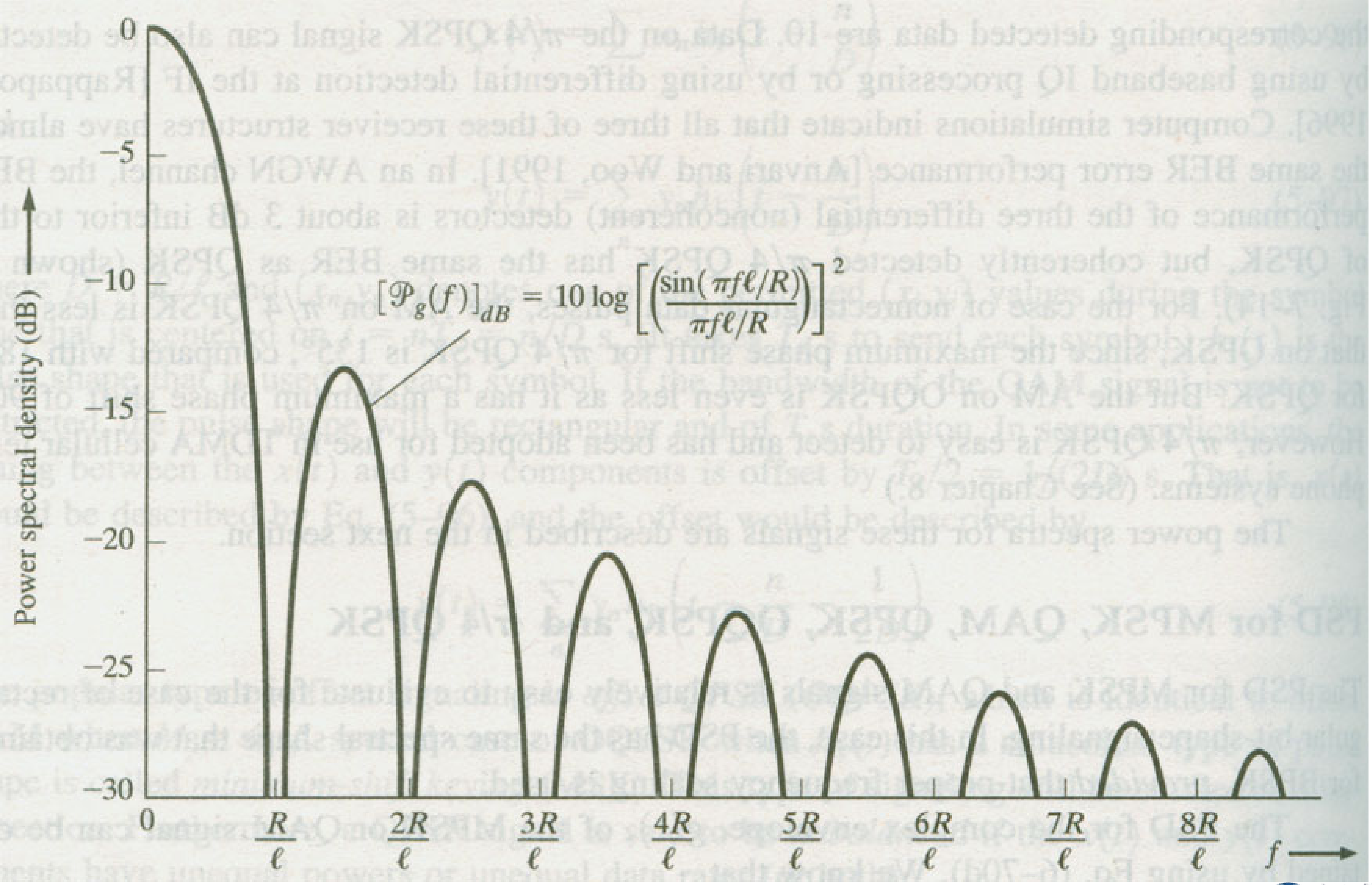
where $K = C\ell T_b$

thus, the **PSD of the MPSK or QAM signal** is obtained:

$$P_s(f) = \frac{1}{4}\left[p_g(f - f_c) + p_g(f + f_c)\right]$$

Multilevel Modulation

PSD for MPSK,QAM,QPSK,OQPSK and π /4 QPSK



Multilevel Modulation

- The spectral sidelobes can be eliminated if **raised cosine filtering** is used

- But the raised cosine filter has the **disadvantage** of introducing AM on MPSK signals (and modifying the AM on QAM signals)

Multilevel Modulation

Spectral efficiency for MPSK,QAM,QPSK,OQPSK and π /4 QPSK with raised cosine filtering

- The absolute transmission bandwidth of the QAM signal with raised cosine filtered pulses is:

$$B_T = \frac{1+r}{\ell} R$$

- The spectral efficiency is:

$$\eta = \frac{R}{B_T} = \frac{\ell}{1+r} = \frac{\ln M}{(1+r)\ln 2}$$

- The result also holds for MPSK.

Multilevel Modulation

Spectral efficiency for MPSK,QAM,QPSK,OQPSK and π /4 QPSK with raised cosine filtering

- **The result is important because it tells us how fast we can signal for a prescribed bandwidth.**

Table 5–6 SPECTRAL EFFICIENCY FOR QAM SIGNALING WITH RAISED COSINE-ROLLOFF PULSE SHAPING (Use $M = 4$ for QPSK, OQPSK, and $\pi/4$ QPSK signaling)

Number of Levels, M (symbols)	Size of DAC, ℓ (bits)	$\eta = \frac{R}{B_T}\left(\frac{\text{bit/s}}{\text{Hz}}\right)$					
		$r = 0.0$	$r = 0.1$	$r = 0.25$	$r = 0.5$	$r = 0.75$	$r = 1.0$
2	1	1.00	0.909	0.800	0.667	0.571	0.500
4	2	2.00	1.82	1.60	1.33	1.14	1.00
8	3	3.00	2.73	2.40	2.00	1.71	1.50
16	4	4.00	3.64	3.20	2.67	2.29	2.00
32	5	5.00	4.55	4.0	3.33	2.86	2.50

5.11 Minimum-shift keying MSK and GMSK

Background

In practice, the communication system need to

- conserve efficiently bandwidth
- improve the efficiency of power supply.

Minimum-shift keying is another bandwidth conservation technique that has been developed. Its advantage of producing a constant-amplitude signal and , consequently, can be amplified with Class C amplifiers without distortion

MSK & GMSK

Properties of MSK

1) continuous binary FSK;

2) two signals are orthogonal over the bit interval;

3) modulation index being minimum, $h=0.5$

4) being identical with OQPSK when the data pulse shape is sinusoidal type.

MSK & GMSK

Now we will demonstrate that the MSK signal (which is h=0.5 continuous-phase FSK) is a form of OQPSK signaling with sinusoidal pulse shaping.

complex envelope for MSK

$$g(t) = A_c e^{j\theta(t)} = A_c e^{j2\pi\Delta F\int_0^t m(\lambda)d\lambda} = A_c e^{\pm j\frac{\pi}{2T_b}t} \quad (0 < \mathrm{t} < \mathrm{T_b})$$

or $$g(t) = x(t) + jy(t) \quad (0 < \mathrm{t} < \mathrm{T_b})$$

$$x(t) = A_c \cos(\pm 1 \times \frac{\pi \mathrm{t}}{2\mathrm{T_b}}), \quad (0 < \mathrm{t} < \mathrm{T_b})$$

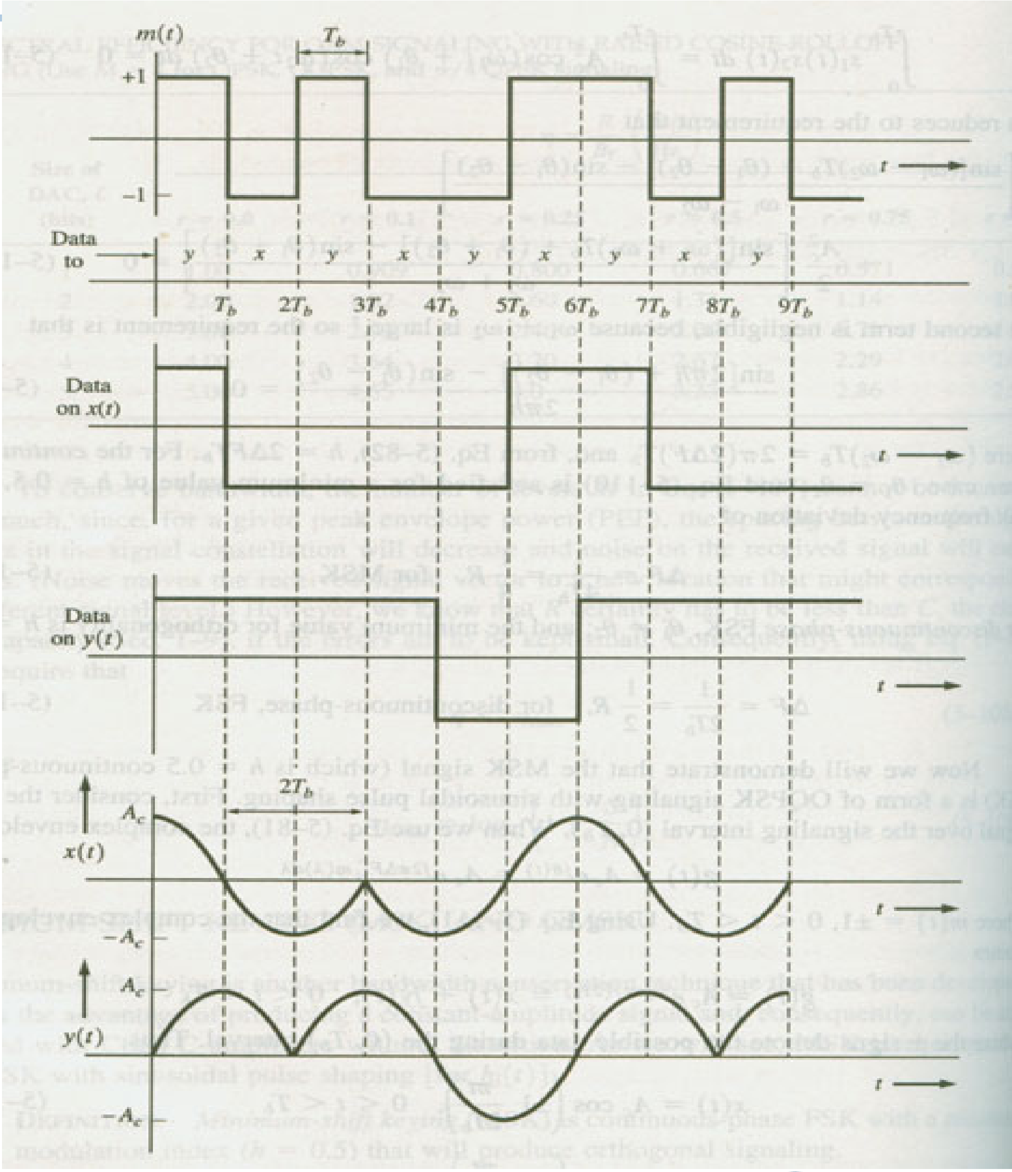
$$y(t) = A_c \sin(\pm 1 \times \frac{\pi \mathrm{t}}{2\mathrm{T_b}}), \quad (0 < \mathrm{t} < \mathrm{T_b})$$

MSK & GMSK

The MSK signaling is:

$$s(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

The binary data of ***m*(t)** alternately modulate the *x*(t) and *y*(t) components, and the pulse shape for the *x*(t) and *y*(t) symbols is a sinusoid.



MSK & GMSK

- ***Type I MSK*:**

  Basic pulse shape [for both x(t) and y(t)] alternates between a positive and a negative half-cosinusoid

- ***Type II MSK:***

  Basic pulse shape is always a positive half cosinusoid

PSD for MSK

The PSD for the complex envelope is:

$$\mathcal{P}_g(f)=\mathcal{P}_x(f)+\mathcal{P}_y(f)=2\mathcal{P}_x(f)$$

Let the pulse width is $2T_b$, this PSD becomes:

$$\mathcal{P}_g(f)=\frac{2}{2T_b}\left|F(f)\right|^2$$

For the MSK half-cosinusoidal pulse shape, we have:

$$f(t)=\begin{cases} A_c\cos\left(\dfrac{\pi t}{2T_b}\right) & |t|<T_b \\ 0 & elsewhere \end{cases}$$

MSK & GMSK

and its Fourier transform is:

$$F(f) = \frac{4A_c T_b \cos 2\pi T_b f}{\pi\left[1-(4T_b f)^2\right]}$$

Thus, **the PSD for the complex envelope for an MSK signal** is:

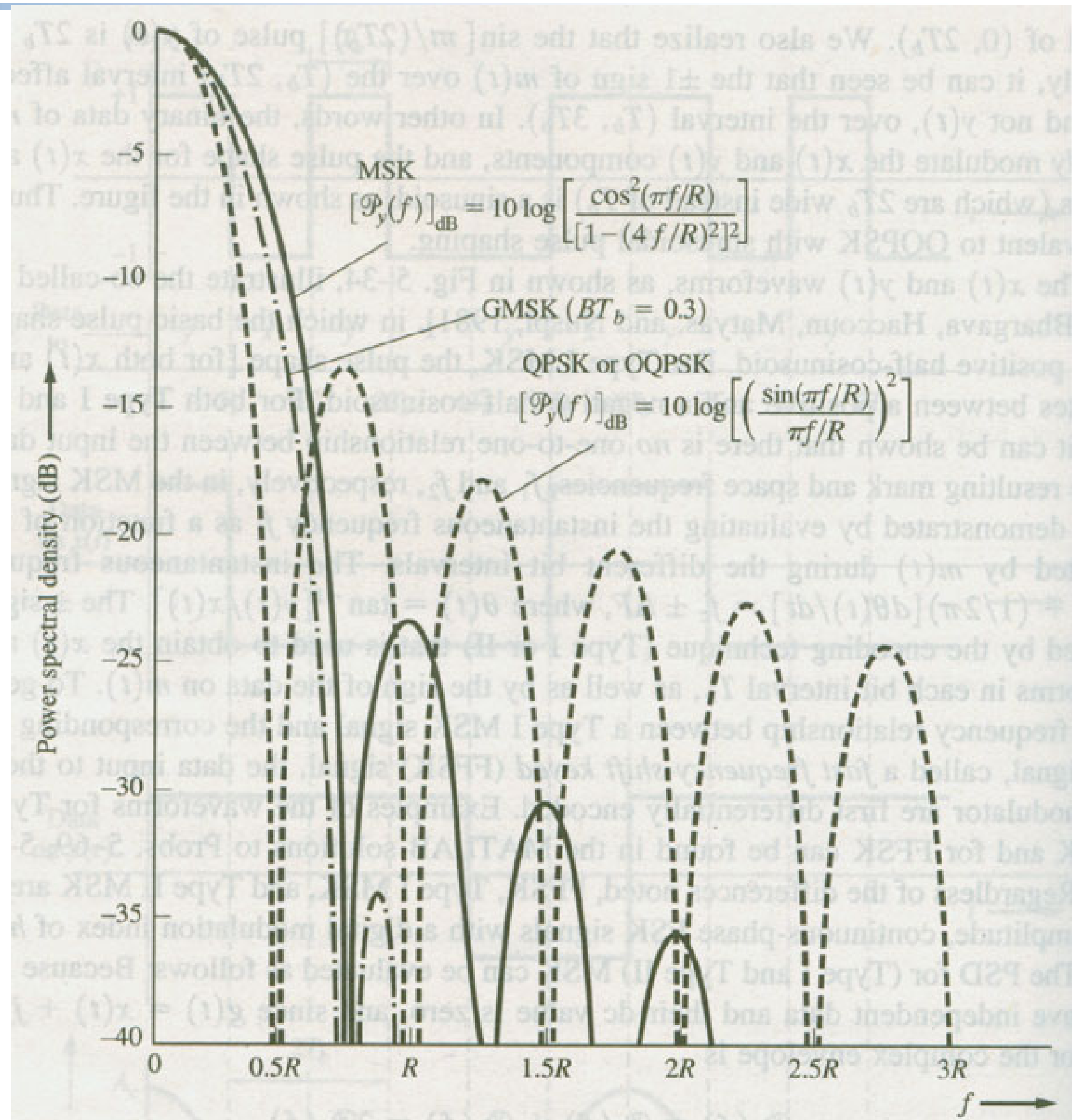
$$\mathcal{P}_g(f) = \frac{16A_c^2 T_b \cos^2 2\pi T_b f}{\pi^2\left[1-(4T_b f)^2\right]^2}$$

MSK & GMSK

❖ PSD for MSK

PSD for complex envelope of MSK (solid-line curve)

The PSD for MSK is readily obtained by translating this spectrum up to the carrier frequency.



GMSK

Gaussian-filtered MSK (GMSK)

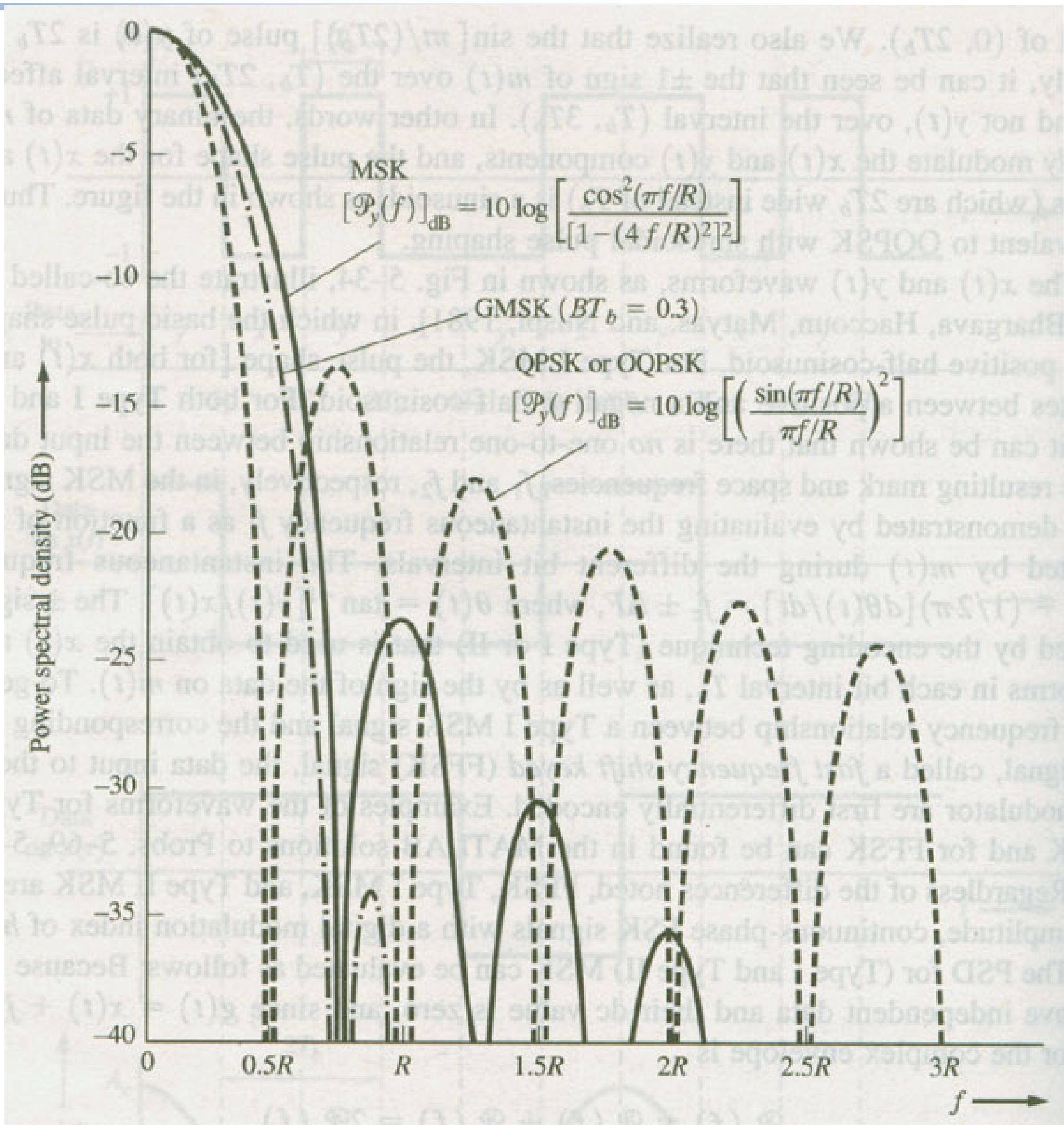
- Before the data are frequency modulated onto the carrier, the data are filtered by a filter having a Gaussian-shaped frequency response characteristic.
- The transfer function of the Gaussian low-pass filter is

$$H(f) = e^{-\left[(f/B)^2 (\ln 2/2)\right]}$$

MSK & GMSK

GMSK

This filter reduces the spectral sidelobes on the transmitted MSK signal.



GMSK

Properties of GMSK

1) **Low spectral sidelobes on the transmitted MSK signal**

2) **constant envelope**

- **GMSK can be detected either coherently or incoherently**

- **GMSK with BT_b=0.3 is the modulation format used in GSM cellular telephone system.**

Summary

Binary modulated Bandpass Signaling

- OOK
- BPSK
- FSK

Multilevel Modulated Bandpass Signaling

- MPSK & QPSK
- QAM
- OQPSK
- $\pi/4$ QPSK

MSK & GMSK

Summary

Comparison of the spectral efficiencies of various types of digital signal

Type of Signal	Spectral Efficiency, $\eta = \frac{R}{B_T}\left(\frac{\text{bits/s}}{\text{Hz}}\right)$ Null-to-Null Bandwidth	30-dB Bandwidth
OOK and BPSK	0.500	0.052
QPSK, OQPSK, and $\pi/4$ QPSK	1.00	0.104
MSK	0.667	0.438
16 QAM	2.00	0.208
64 QAM	3.00	0.313